



ECE 495 – Electromagnetic Warfare and Survivability

Block 1: Radar Fundamentals

Lesson 3: The Radar Range Equation

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L1 hinted: $R_{\max} \propto \sigma^{1/4}$.

L2 gave us one-way FSPL: $L_{\text{fs}} \propto (R/\lambda)^2$.

Today: the full radar range equation that ties them together — the central equation of the course.

By the end of this lesson, you will be able to:

- Derive the **radar range equation** from one-way propagation and a passive target
- Identify each term and what it represents physically
- Compute $R_{\max}$ for a notional engagement
- Connect L1's fourth-power law back to the full equation

From One-Way to Two-Way



L2 was a transmitter and a receiver. Radar adds a **passive target** in between.

- One-way path: one factor of $1/R^2$ from spherical spreading
- Radar path: **two** factors of $1/R^2$ — out and back
- The target itself acts as a passive antenna with effective area σ

Two trips through free space, one bounce off the target.

Deriving the Radar Range Equation



Step 1. Tx radiates P_t with gain G_t . Power density at the target:

$$S_t = \frac{P_t G_t}{4\pi R^2}$$

Step 2. Target with RCS σ re-radiates:

$$P_{\text{ref}} = S_t \sigma$$

Step 3. Power density back at the radar (second $1/R^2$):

$$S_r = \frac{P_{\text{ref}}}{4\pi R^2} = \frac{P_t G_t \sigma}{(4\pi)^2 R^4}$$

Step 4. Receiver effective aperture $A_e = G_r \lambda^2 / (4\pi)$ captures:

$$P_r = S_r A_e$$

The Radar Range Equation



$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4}$$

Detection threshold: $P_r \geq S_{\min}$. Solving at equality:

$$R_{\max} = \left[\frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 S_{\min}} \right]^{1/4}$$

- Every lever the radar designer pulls lives in the numerator
- Every term the target controls (just σ) lives in the numerator too
- $R_{\max} \propto (\text{everything})^{1/4}$ — the fourth-power penalty

Each Term Has a Cost



Term	Who controls it	Engineering cost
P_t	Radar designer	Prime power, cooling, hardware
G_t, G_r	Radar designer	Antenna aperture, mechanical size
λ	Mission planner	Band selection (atmosphere, hardware)
σ	<i>Aircraft designer</i>	Shape, materials, EMCON
$S_{\min}$	Radar designer	Receiver design, BW, integration

- Every parameter pays its dB at $\frac{1}{4}$ efficiency in $R_{\max}$
- σ is the only lever the aircraft fights for directly

$$R_{\max} = K \cdot \sigma^{1/4}, \quad K \equiv \left[\frac{P_t G_t G_r \lambda^2}{(4\pi)^3 S_{\min}} \right]^{1/4}$$

- K is everything except σ — fixed once the threat is chosen
- Halving $R_{\max}$ takes σ down by **12 dB** — the cost of LO
- L1's notional $K = 100 \text{ km} / \text{m}^{1/2}$ was the radar range equation in disguise

L1's fourth-power law was the radar equation hiding in one constant.

Worked Example: S-band Acquisition vs a 1 m² Fighter

Given: S-band acquisition (ACQ) radar engaging a legacy fighter.

$$P_t = 1 \text{ MW} \Rightarrow 60 \text{ dBW}$$

$$\sigma = 1 \text{ m}^2 \Rightarrow 0 \text{ dBsm}$$

$$G_t = G_r = 30 \text{ dBi}$$

$$S_{\min} = -130 \text{ dBm} \Rightarrow -160 \text{ dBW}$$

$$\lambda = 0.1 \text{ m} \Rightarrow 20 \log_{10} \lambda = -20 \text{ dB-m} \quad (4\pi)^3 \approx 33 \text{ dB}$$

Working in dB:

$$10 \log_{10}(R_{\max}^4) = P_t + G_t + G_r + 20 \log \lambda + \sigma - 33 - S_{\min}$$

$$= 60 + 30 + 30 - 20 + 0 - 33 + 160 = 227 \text{ dB}$$

$$\Rightarrow 10 \log_{10} R_{\max} = 56.75 \text{ dB-m} \Rightarrow R_{\max} \approx 474 \text{ km}$$

Now swap in the B-21 ($\sigma \approx -30 \text{ dBsm}$): $R_{\max}$ drops by $(10^{-3})^{1/4} = 0.178 \Rightarrow R_{\max} \approx 84 \text{ km}$.

Today's interactive demo.

- Compute $R_{\max}$ for the worked example
- Sweep σ across -50 to $+10$ dBsm and plot $R_{\max}$
- Overlay L1's $K \cdot \sigma^{1/4}$ approximation
- Verify the 12 dB rule: how much σ reduction halves $R_{\max}$?

Companion script: `L3_RadarRangeEquation.m`

In L4 we add the time dimension: pulse width, PRF, and ambiguity.

Quick Exercise / Type-along: Four Levers



Type this with me. One anonymous function, four one-line experiments.

```

Rmax = @(Pt,Gt,Gr,lam,sig,Smin) ((Pt*Gt*Gr*lam^2*sig)/((4*pi)^3*Smin))^(1/4);
Pt = 1e6; Gt = 1e3; Gr = 1e3; lam = 0.1; sig = 1; Smin = 1e-16;
R0 = Rmax(Pt,Gt,Gr,lam,sig,Smin);
fprintf('Baseline: R_max = %.0f km\n\n', R0/1000);
name = {'RCS / 16', 'Power x 4', 'Gains x 2 each', 'S_min / 4'}; R = zeros(1,4);
R(1) = Rmax(Pt,Gt,Gr,lam,sig/16,Smin);
R(2) = Rmax(4*Pt,Gt,Gr,lam,sig,Smin);
R(3) = Rmax(Pt,2*Gt,2*Gr,lam,sig,Smin);
R(4) = Rmax(Pt,Gt,Gr,lam,sig,Smin/4);
dTerm = 10*log10([1/16 4 4 4]);
for k = 1:4
    fprintf('%-15s dTerm = %+6.2f dB    R_max = %4.0f km    dR_max = %+5.2f dB\n', ...
            name{k}, dTerm(k), R(k)/1000, 10*log10(R(k)/R0));
end
  
```

What do you notice?

Key ideas:

- $P_r \propto P_t G_t G_r \lambda^2 \sigma / R^4$ — the central equation of B1
- $R_{\max} \propto (\text{everything})^{1/4}$ — fourth-power penalty for everyone
- σ is the airframe's lever; $S_{\min}$ is the receiver designer's

Next lesson:

- **L4: Pulse radars** — PW, PRF, duty cycle, and range ambiguity
- Pre-flight quiz for L4 due before class